Compression modes in nuclei: microscopic models with Skyrme interactions

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The isoscalar giant monopole resonances (ISGMR) and giant dipole resonances (ISGDR) in medium-heavy nuclei are investigated in the framework of HF+RPA and HF-BCS+QRPA with Skyrme effective interactions. It is found that pairing has little effect on these modes. It is also found that the coupling of the RPA states to 2p-2h configurations results in about (or less than) 1 MeV shifts of the resonance energies and at the same time gives the correct total widths. For the ISGMR, comparison with recent data leads to a value of nuclear matter compression modulus close to 215 MeV. However, a discrepancy between calculated and measured energies of the ISGDR in ^{208}Pb is found and remains an open problem.

Introduction

The determination of the nuclear incompressibility K is still a matter of debate, despite a remarkable number of works on the subject[1]. In the present contribution, we present self-consistent calculations of the nuclear collective modes associated with a compression and expansion of the nuclear volume, namely the isoscalar giant monopole and dipole resonances (ISGMR and ISGDR, respectively). In fact, we share the point of view that the most reliable way to extract information on K is to perform that kind of calculations, having as the only phenomenological input a given effective nucleon-nucleon interaction, and choose the value of K corresponding to the force which can reproduce the experimental properties of the compression modes in finite nuclei.

The ISGMR, or "breathing mode", is excited by the operator $\sum_{i=1}^{A} r_i^2$ and it has been identified in many isotopes along the chart of nuclei already two decades ago. However, this systematics has never allowed an unambigous determination of K[1]. This was one of the motivations for the recent experimental program undertaken at the Texas A& M Cyclotron Institute, which has allowed the extraction of experimental data for the ISGMR of better quality as compared to the past, by means of the analysis of the results of inelastic scattering of 240 MeV α-particles. We refer to other contributions in these proceedings for reports on these experimental data[2]. Monopole strength functions turn out to be quite fragmented for nuclei lighter than ^{90}Zr. For nuclei like ^{208}Pb, ^{144}Sm, ^{116}Sn and ^{90}Zr, however, one is able to identify a single peak which, together with a high-energy extended tail, exhausts essentially all the monopole Energy Weighted Sum Rule (EWSR). These medium-heavy nuclei are, therefore, those suited for the extraction of information about the nuclear incompressibility and we concentrate ourselves on them in the present work.

The ISGDR is excited by the operator $\sum_i r_i^3 Y_{10}$ and corresponds to a compression of the nucleus along a definite direction, so that it has been called sometimes the "squeezing mode". Although some first indication about the energy location of this resonance dates back to the beginning of the eighties, a more clear indication about its strength distribution in ^{208}Pb has been reported only recently[3]. Measurements have been done also for other nuclei, namely ^{90}Zr, ^{116}Sn and ^{144}Sm (as in the case of the giant monopole resonance). There is some expectation that the study of this mode can help to shed some light on the problem of nuclear incompressibility. Actually, at first sight this compressional mode seems to provide us with a new problem. A simple assumption like the scaling model (illustrated for the present purposes in Ref.[4]) would lead to two different values of the finite-nucleus incompressibility K_A if applied to the ISGMR and the ISGDR with the input of their experimental energies. The hydrodynamical model gives two results which are closer[4] but which still make us wonder about the validity of methods based on extracting K_A and extrapolating it to large values of A, for the determination of K. This points again to the necessity of reliable microscopic calculations of the compressional modes, in order to reproduce the experimental data and extract the value of K from the properties of the force which is used.

Our calculations are performed within the framework of self-consistent Hartree-Fock (HF) plus Random-Phase Approximation (RPA). We use effective forces of Skyrme type[5−7] and we look at their predictions for the properties of ISGMR and ISGDR. The parametrizations we employ span a large range of values for K (from 200 MeV to about 350 MeV). In particular, we focus mainly on two original aspects: firstly, we look at the effects of pairing correlations in open-shell nuclei; secondly, we study if the picture obtained at mean-field level is altered by the inclusion of the coupling of the giant resonances to

more complicated nuclear configurations. This inclusion is necessary if one wishes to understand theoretically all the contributions to the resonance width and may shift the resonance centroid.

About the first aspect, it is well known that pairing correlations are important in general to explain the properties of ground states and low-lying excited states in open-shell nuclei. Since we wish to see how these correlations affect in particular the compressional modes, we take them into account by extending the HF-RPA approach to a quasi-particle RPA (QRPA) on top of a HF-BCS calculation.

About the second aspect, we recall that if we start from a description of the giant resonance as a superposition of one particle-one hole (1p-1h) excitations, in their damping process we must take care of the coupling with states of 2p-2h character. They are in fact known to play a major role and give rise to the spreading width $\Gamma^\downarrow$ of the giant resonance which is usually a quite large fraction of the total width. Within mean field theories, only the width associated with the resonance fragmentation (Landau width) and the escape width $\Gamma^\uparrow$ are included (the latter, provided that 1p-1h configurations with the particle in the continuum are considered). In the past, we have developed a theory in which all the contributions to the total width of giant resonances are consistently treated and we have obtained satisfactory results when applying it to a number of cases. In particular, we will recall what has been obtained[8] for the case of the ISGMR in ^{208}Pb. We also report about a new calculation for the ISGDR in the same nucleus.

Formalism: a brief survey

For all nuclei we consider, we solve the HF equations on a radial mesh and, in the case of the open-shell isotopes, we solve HF-BCS equations. A constant pairing gap Δ is introduced (for neutrons in the case of ^{116}Sn and for protons in the case of ^{90}Zr and ^{144}Sm), and at each HF iteration the quasi-particle energies, the occupation factors and the densities to be input at the next iteration are determined accordingly. Δ is obtained from the binding energies of the neighboring nuclei[9]. The states included in the solution of the HF-BCS equations are those below a cutoff energy given by $\lambda_{HF} + 8.3$ MeV (λ_{HF} being the HF Fermi energy), in analogy with the procedure of Ref.[10].

Using the above self-consistent mean fields we work out the RPA or QRPA equations (respectively on top of HF or HF-BCS), in their matrix form. Discrete positive energy states are obtained by diagonalizing the mean field on a harmonic oscillator basis and they are used to build the 1p-1h (or 2 quasi-particles) basis coupled to $J^\pi=0^+$ or 1^-. The dimension of this basis is chosen in such a way that more than 95% (typically 97-99%) of the appropri-

ate EWSR is exhausted in the RPA or QRPA calculation. More details, especially on the way the QRPA equations are implemented, will be given in Ref.[11].

As mentioned in the previous section, in the case of ^{208}Pb we perform also calculations that go beyond this simple discrete RPA. This is done along the formalism described in Ref.[12], which is recalled here only very briefly.

We label by Q_1 the space of discrete 1p-1h configurations in which the RPA equations are solved. To account for the escape width $\Gamma^\uparrow$ and spreading width $\Gamma^\downarrow$ of the giant resonances, we build two other orthogonal subspaces P and Q_2. The space P is made of particle-hole configurations where the particle is in an unbound state orthogonal to all the discrete single-particle levels; the space Q_2 is built with the configurations which are known to play a major role in the damping process of giant resonances: these configurations are 1p-1h states coupled to a collective vibration. Using the projection operator formalism one can easily find that the effects of coupling the subspaces P and Q_2 to Q_1 are described by the following effective Hamiltonian acting in the Q_1 space:

$$
\begin{aligned}
\mathcal{H}(E) \equiv Q_1 H Q_1 \quad &+ \quad W^\uparrow(E) + W^\downarrow(E) \\
= Q_1 H Q_1 \quad &+ \quad Q_1 H P \frac{1}{E - PHP + i\epsilon} P H Q_1 \\
&+ \quad Q_1 H Q_2 \frac{1}{E - Q_2 H Q_2 + i\epsilon} Q_2 H Q_1,
\end{aligned}
$$

where E is the excitation energy. For each value of E the RPA equations corresponding to this effective, complex Hamiltonian $\mathcal{H}(E)$ are solved and the resulting sets of eigenstates enable us to calculate all relevant quantities, in particular the strength function associated with a given operator. To evaluate the matrix elements of $W^\downarrow$, we calculate the collective phonons with the same effective interaction used for the giant resonance we are studying (within RPA), and we couple these phonons with the 1p-1h components of the giant resonance by using their energies and transition densities.

Results for the isoscalar monopole resonance

As recalled in the introduction, Youngblood et al.[2] have recently measured the ISGMR strength distribution with fairly good precision, in the nuclei ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb. In their work, they also compare the experimental centroid energies with the calculations of Blaizot et al.[13] performed by using RPA and employing the finite-range Gogny effective interaction: a value of the nuclear incompressibility $K = 231$ MeV is deduced. In the following, we denote as centroid energy the ratio $E_0 \equiv m_1/m_0$ (m_0 and m_1 being the non-energy-weighted and energy-weighted sum rules, respectively).

If we try to compare the experimental values with calcu-

lations done at the same RPA level but using the zero-range Skyrme effective interactions, we can infer a different conclusion with respect to the value of K. Among the Skyrme type interactions, the parametrization which gives probably the best account of the experimental centroid energies in the nuclei studied by the authors of Ref.[2], is the SGII force[6]. The results are shown in Table 1. The force SGII is characterized by a value of the nuclear incompressibility $K = 215$ MeV: since it reproduces very well the ISGMR centroid energy in ^{208}Pb, and it slightly overestimates those in the other isotopes, one would conclude that K is of the order of or slightly less than 215 MeV.

This conclusion is inferred by means of simple RPA. It is of course legitimate to wonder if calculations beyond this simple approximation could lead to different values of the nuclear incompressibility. We first consider the effect of pairing correlations. In the case of ^{116}Sn, the centroid energy of 17.18 MeV obtained with the SGII force in RPA, becomes 17.19 MeV if one turns to QRPA. A very small shift is found also when other forces are used (for instance, with the recently proposed SLy4 force[7], one obtains 17.51 MeV and 17.59 MeV for RPA and QRPA, respectively) and when other nuclei are considered. In general, although we know that pairing correlations play a crucial role not only to explain the ground-state of open-shell nuclei but also their low-lying excited states, it appears that they do not affect so much the giant resonances like the ISGMR (or ISGDR, anticipating results of the next section) which lie at relatively high excitation energy compared to the pairing gap Δ. Civitarese *et al.*[14] found also small shifts (of the order of 100-150 keV) for the ISGMR and ISGQR when pairing correlations are taken into account: this shift is larger than that obtained in the present work, but it is the result of a different (non self-consistent) model.

The present conclusion for the nuclear incompressibility is therefore similar to that obtained by Hamamoto *et al.*[15], since they find that the Skyrme interaction which provides the best results for the ISGMR is the SKM* parametrization and this is very similar to SGII (the associated nuclear incompressibility being 217 MeV). Our study, however, is done in a more general framework since we have analyzed also the role of pairing correlations.

If we finally consider the results of calculations beyond mean field[8] (which include not only the continuum coupling but also the coupling with the 2p-2h type states) performed for ^{208}Pb we find that it is also possible to reproduce rather well the total width of the ISGMR, which is around 3 MeV. This width is actually in large part a consequence of fragmentation (or Landau damping): at least three states share, at the level of RPA, the resonance strength, but continuum as well as 2p-2h couplings are able to give to each peak the correct width so that the overall lineshape coincides with the experimental findings. We stress that the coupling with the 2p-2h type

states is also responsible for a downward shift of the IS-GMR centroid and peak energies, which is of the order of 0.5 MeV. One may argue that this affects the extraction of the value of K from theoretical calculations. Actually, since the value of K associated with a given force is obtained by a calculation of nuclear matter at the mean field level, it is legitimate to draw conclusions about K from the comparison with the experiment of the ISGMR results for finite nuclei obtained again at the mean field level. But the fact that a given force is able to account for the ISGMR linewidth enforces our confidence about its reliability. And it would be of course legitimate either, to compare the centroid energies obtained after 2p-2h coupling with experiment provided the value of K associated with the force is calculated by including the same couplings at the nuclear matter level. No such calculations in nuclear matter have been done so far, to our knowledge.

Table 1. Experimental and theoretical values of the centroid energies $E_0 \equiv m_1/m_0$ for the ISGMR and IS-GDR. The theoretical values are obtained with the Skyrme-RPA approach, using the SGII interaction. All values are in MeV.

	ISGMR		ISGDR	
	Exp.	Theory	Exp.	Theory
^{90}Zr	17.9	19.1	26.2	27.1
^{116}Sn	16.0	17.2	23.0	26.3
^{144}Sm	15.3	16.2	24.2	25.4
^{208}Pb	14.2	14.1	20.3	24.1

Results for the isoscalar dipole resonance

A peculiar feature of calculations of this giant resonance is the appearance of a spurious state in the calculated spectrum. When diagonalizing the RPA matrix on a 1^- basis, we expect to see among all states the spurious state at zero energy corresponding to the center-of-mass motion and we expect as well that it exhausts the whole strength associated with the operator $\sum_i r_i Y_{10}$. Due to a lack of complete self-consistency (some part of the residual interaction, like the two-body spin-orbit and Coulomb forces, are usually neglected in the RPA because their effect should be rather small) and to numerical inaccuracies, this is not the case. The spurious state comes out in practice at finite energy and its wave function does not overlap completely with that of the exact center-of-mass motion: as a consequence, the remaining RPA eigenstates are not exactly orthogonal to the true spurious state, and their spurious component must be projected out. This is not difficult if RPA is done in the discrete p-h space. In any case, it can be shown[16] that this projection procedure is equivalent to replacing the $\sum_i r_i^3 Y_{10}$ operator with $\sum_i (r_i^3 - \eta r_i) Y_{10}$, η being $\frac{5}{3} \langle r^2 \rangle$. Once this projection is done, we find that a substantial amount of $r^3 Y_{10}$ strength still remains in the 10 - 15 MeV region, in addition to the strength in the

20 - 25 MeV region. The data do not show any strength in the lower region, i.e., experimental centroid energies correspond to the energy region above 15 MeV. Therefore, to have a meaningful comparison with experiment we will refer from now on to theoretical centroid energies calculated in the interval 15 - 40 MeV.

In Table 1 we show the ISGDR centroid energies obtained with the SGII force. Especially in the case of ^{208}Pb and ^{116}Sn, it can be noticed that RPA calculations tend to overestimate the value of the centroid energy, the discrepancy being less severe in the other two cases. One may wonder if this is a special feature of SGII, although this force has been said to behave rather well for the monopole case. Fig. 1 shows that this is not the case: the centroid energies obtained in RPA with a number of different Skyrme parametrizations are plotted as a function of their incompressibility K, and it can be noticed that all forces systematically overestimate the experimental values of the centroid energies. Gogny interactions have also been used to study the ISGDR in this and other nuclei[17], but they also predict too large centroid energies. The same can be said about relativistic models like relativistic RPA[18] or time-dependent relativistic mean field[19]. We may conclude that the case of the ISGDR in ^{208}Pb is a kind of exception among the giant resonances studied within the self-consistent HF-RPA approach, as usually one never finds such large discrepancies between theory and experiment ($\sim$ 4-5 MeV).

We finally address the question whether the coupling with more complicated configurations, which has been seen to be responsible of a downward shift of the resonance energy, can diminish this discrepancy in the case of ^{208}Pb. We have done for the ISGDR a calculation of the type described above in the monopole case. The resulting strength function, which includes continuum and 2p-2h coupling, is depicted in Fig. 2. One can see that the total width of the resonance is quite large, and the theoretical value of about 6 MeV compares well with the experimental result which is about 7 MeV[20]. Although the resonance lineshape is accounted for by theory, the downward shift with respect to the RPA result is only about 1 MeV. The fundamental problem why the ISGDR energy in ^{208}Pb cannot be reproduced by theoretical models still remains.

Conclusion

In this paper, we have considered the isoscalar monopole and dipole resonances in a number of nuclei and we have tried to reproduce their properties by means of HF-RPA, or QRPA on top of HF-BCS, or more sophisticated approach which takes care of the continuum properly and of the coupling with nuclear configurations which are more complicated than the simple 1p-1h. In general, we have found that the effect of pairing correlations is quite small

as these resonances lie at high energy with respect to the pairing gap Δ.

Concerning the RPA results, the situation looks different in the case of monopole and dipole. In the former case, the Skyrme-type force SGII is able to reproduce well the centroid energy in ^{208}Pb, and it slightly overestimates this energy in other medium-heavy nuclei which have been measured accurately in recent experiments. This would allow us to extract a value of the nuclear incompressibility around 215 MeV. In the case of the ISGDR, however, the same Skyrme force overpredicts this centroid energy in ^{208}Pb by about 4 MeV. Other parametrizations of Skyrme type cannot do better, and the problem is not solved if one turns to Gogny interactions or to relativistic models. Therefore, although in other nuclei this discrepancy between theory and experiment can be less than in the case of ^{208}Pb, we can say that the "squeezing mode", which could be taken as a further probe of the nuclear incompressibility besides the well-known "breathing" monopole oscillation, is challenging us with a new problem.

Calculations beyond mean field do not change substantially our conclusions about the centroid energies. However, we stress that these calculations are necessary for a proper account of the giant resonances lineshape, and in fact in our case they have been able to reproduce the total width of the ISGMR in ^{208}Pb, and also of the ISGDR although its centroid is overestimated.

Fig. 1. RPA centroid energies of ISGDR calculated in ^{208}Pb with various Skyrme interactions, as a function of compression modulus K.

Fig. 2. Calculated ISGDR strength distribution of operator $(r^3 - \eta r)Y_{10}$ including damping effects of particle-hole-plus-phonon coupling (see text). The nucleus is ^{208}Pb.

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